Performance Evaluation of Mobile Radio Slotted ALOHA with Smart Antennas

WK Fung, M Hamdi and R.D. Murch
Hong Kong University of Science and Technology
Clear Water Bay, Kowloon, Hong Kong
hamdi@cs.ust.hk eermurch@ee.ust.hk

Abstract— **We report an investigation into the effect of using smart antennas on the performance of the slotted ALOHA protocol with capture in a mobile communications environment with Rayleigh and Log-normal fading. Our results demonstrate that by using smart antennas, we can achieve higher performance in terms of capture probability and throughput when compared to a conventional antenna system using the slotted ALOHA protocol.**

I. INTRODUCTION

To allow relatively uncordinated users to share a common wireless channel, in wireless LANs, the slotted ALOHA random access protocol has been proposed as a possible solution [1], [2], [3]. In the original version of slotted ALOHA it is assumed that whenever more than one packet is transmitted at the same time collisions occur and the information in all the packets are lost. In particular, under this assumption, the maximum system throughput of a slotted ALOHA is limited to just around 36%. However in a mobile wireless environment the "capture effect" can be exploited to allow the strongest signal to be received even in the presence of collisions, and as a result, increases the throughput of packets in the system [4]. An additional method, in the mobile wireless environment, which could also be utilized to overcome lost packets that have collided is the use of smart antennas. Smart antennas provide angular filtering of the incoming wireless signals and can therefore be exploited to reduce collisions of packets arriving from different directions and hence increase the overall throughput of the system [5] - [9].

Previous research efforts on wireless slotted ALOHA have analyzed the capture effect in wireless environments and also the use of diversity on the system throughput [3], [4], [10], [11]. However, to the best of our knowledge, few results are available on the effect of smart antennas on mobile wireless slotted ALOHA.

In this paper we present an assessment of the throughput and capture probability of a mobile wireless slotted ALOHA system in which smart antennas are utilized. In our analyses we assume a switched beam smart antenna in which the effect of the number of beams, m, and the beam-width, α, is investigated. We assume the signal propagation experiences Rayleigh and log-normal fading, and attenuation is dependent on distance. The key in our analysis however is utilizing the angular dependence of the smart antennas to provide angular filtering of the wireless signals. We also calculate the capture probability, the throughput when using multiple receivers and address the stability issue by computing the capture probability when the number of users, n, tends to infinity.

II. SYSTEM MODEL

We assume users are distributed at random in a circular cell whose radius is normalized to unity. The switched beam antenna has m beams and is shown in figure 1. We assume idealized beams so that, in figure 1, for $m = 2$, the upper beam will receive signals in sector A but not in sector B and the lower beam will receive those signals in sector B only but not in sector A. Slotted ALOHA is used as the multiple access scheme so that when a user has a new packet, it is sent in the next time slot and will not consider whether other users are also sending packets or not. If transmission fails in that time slot, it will be re-transmitted after a random number of time slots later. While the user is in this waiting stage, no new packets will be generated.

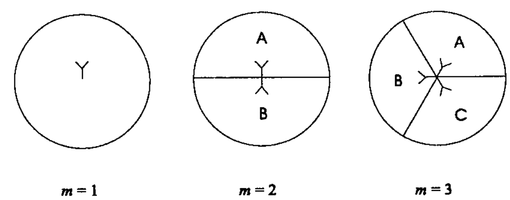

Fig. 1. Plan view for the smart antennas when m =1, 2 and 3

Following [3] propagation is described by means of four effects: attenuation due to the distance r, proportional to $r^{-\eta}$, where η, the power loss law exponent, assumes values between 2 and 4, and is typically taken equal to 4 in land mobile wireless environments; shadowing, described by means of a log-normal r.v.; Rayleigh fading, which causes the instantaneous envelope of the received signal to be Rayleigh distributed; and angular filtering depends on the angular position θ of the user and is specified by the function, $f(\theta)$, which is the pattern of a particular beam in the switched beam antenna system.

With these assumptions, the received power from a mo-

0-7803-5668-3/99/$10.00

bile at location r and θ can be expressed as follows [3], [9], [12], [13]:

$$P_R = R^2 e^{\xi} K r^{-\eta} P_T f(\theta), \tag{1}$$

where R is Rayleigh distributed with unit power, e^{ξ} accounts for the shadowing (ξ is Gaussian with zero mean and variance σ^2), $Kr^{-\eta}$ is the deterministic loss law, P_T is the transmitted power and the angular filtering characteristics of the switched beam antenna are denoted, $f(\theta)$, and follows an idealized pattern specified by:

$$f(\theta) = \begin{cases} 1 & \text{for} \quad -\alpha/2 \le \theta \le \alpha/2 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where α is the angular width of a beam in the switched beam antenna. In general for an idealized switched beam antenna the beamwidth $\alpha = 2\pi/m$ where m is the number of beams. For all users it is assumed K, η and P_T are fixed and the same, whereas R, η and θ are assumed to be independent from user to user and are identically distributed. The log-normal shadowing is expressed in dB instead of natural units, and its standard deviation σ_{dB}, also called dB spread, is related to σ by the relationship $\sigma = (0.1 \log_e 10)\sigma_{dB}$.

The instantaneous signal to noise ratio is expressed as [3]:

$$SNR = \frac{P_{R_0}}{P_N + \sum_{i=1}^{k} P_{R_i}} \tag{3}$$

where the subscript 0 denotes the intended user and the subscript i represents the other user, P_N is the background noise power, and k is the number of interferers in the same slot.

An outage is defined as the event that the SNR falls below a pre-determined threshold. We will assume that when a user experiences an outage its packet is lost, otherwise, it is correctly received. The probability of no outage (i.e., successfully receiving a message) is defined as

$$P_s = P[SNR > b], \tag{4}$$

where b is the SNR threshold for successful reception. To focus on the multiple access, in what follows the effect of noise will be neglected (i.e., $P_N = 0$). From equation (3), we have, at distance r_0 from the base station and angle θ_0 in the presence of k interferers:

$$\begin{aligned} P_s(r_0,\theta_0) &= P[P_{R_0} > b\sum_{i=1}^{k} P_{R_i}] \\ &= P[R_0^2 > b\sum_{i=1}^{k} R_i^2 e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{f(\theta_i)}{f(\theta_0)}] \end{aligned} \tag{5}$$

When conditioned on $\underline{\xi} = (\xi_0, \xi_1, ..., \xi_k)$, $\underline{r} = (r_0, r_1, ..., r_k)$ and $\underline{\theta} = (\theta_0, \theta_1, \theta_2, ..., \theta_k)$, P_s is computed as:

$$\begin{aligned} P_s(r_0,\theta_0|\underline{\xi},\underline{r},\underline{\theta}) &= \int_0^{\infty} da_1 e^{-a_1} ... \int_0^{\infty} da_k e^{-a_k} \\ & \exp(-b\sum_{i=1}^{k} a_i e^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{f(\theta_i)}{f(\theta_0)}) \\ &= \prod_{i=1}^{k} \frac{1}{1+be^{\xi_i-\xi_0}(\frac{r_i}{r_0})^{-\eta}\frac{f(\theta_i)}{f(\theta_0)}}, \end{aligned} \tag{6}$$

To determine the probability of successfully receiving a given user located at (r_0, θ_0) in the presence of k users in a beam of angular width α we assume the statistical distribution of the r_i's all follow a common pdf denoted as $h(r)$ while the distribution of the θ_i's are uniformly distributed within the beam so its pdf is constant with density $1/\alpha$. Therefore in the product of ((6)) all factors are statistically equal. Averaging over ξ_0, (the probability P_s, conditioned on ξ_0 only, is obtained by averaging ((6)) over ξ_i, r_i and θ_i, $i = 1, ..., k$) we obtain the final result:

$$P_s(r_0,\theta_0) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I(\xi_0, r_0, \theta_0)]^k, \tag{7}$$

where

$$\begin{aligned} I(\xi_0, r_0, \theta_0) &= \int_{-\infty}^{\infty} \frac{d\xi}{\sqrt{2\pi}\sigma} e^{-\frac{\xi^2}{2\sigma^2}} \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \frac{d\theta}{\alpha} \\ & \int_0^1 \frac{h(r)dr}{1+be^{\xi-\xi_0}(\frac{r}{r_0})^{-\eta}\frac{f(\theta)}{f(\theta_0)}} \end{aligned} \tag{8}$$

III. CAPTURE PROBABILITY

The capture probability $C_{m,n}$ is defined as the probability of successfully receiving *any* user when n users, located in a cell with m beams, transmit together. To find C_{mn} we denote $P_n(r_0, \theta_0)$ as the probability that the transmission by a test user at a distance r_0 and angle θ_0 is successful, given that n packets were transmitted in a particular beam. Using ((7)) this is given by

$$P_n(r_0,\theta_0) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I(\xi_0, r_0, \theta_0)]^{(n-1)}, \tag{9}$$

where $I(\xi_0, r_0, \theta_0)$ is defined in ((8)). Because all users are in the same beam and we use idealized antenna patterns the θ dependence in ((7)) and ((8)) can be removed so that

$$P_n(r_0) = \int_{-\infty}^{\infty} \frac{d\xi_0}{\sqrt{2\pi}\sigma} e^{-\frac{\xi_0^2}{2\sigma^2}} [I(\xi_0, r_0)]^{(n-1)} \tag{10}$$

where $I(\xi_0, r_0)$ is the same as ((8)) but with θ dependence removed.

We define a beam capture probability C_n as the probability of receiving any one of n packets transmitted by the users in a single beam of a switched beam antenna. This can be calculated from $P_n(r_0, \theta_0)$ by integrating over the beam area and multiplying by n (since successful reception is mutually exclusive) to give

$$C_n = \int_0^1 \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \frac{1}{\alpha} n P_n(r_0, \theta_0) h(r_0) d\theta_0 dr_0 \qquad (11)$$

where $\alpha = 2\pi/m$. From expression ((11)) we know that $P_n(r_0, \theta_0)$ has no θ dependence and deduce that for any number of users in a particular beam the capture probability is the same whatever the beamwidth so that

$$C_n = \int_0^1 n P_n(r_0) h(r_0) dr_0 \qquad (12)$$

To deduce the capture probability $C_{m,n}$ when m beams are used for reception and there are n users transmitting we consider cells with $m = 1, 2, 3, \ldots$ beams in turn.

For $m = 1$, $C_{m,n}$ is equal to C_n because there is only one beam operating which is equivalent to a conventional antenna serving the cell and is the same as in [3].

For $m = 2$, each beam will serve half of the cell, and $C_{m,n}$ represents the capture probability that any one of the two beams receives a message from a user. When there are two beams and n users, there will be say i users (between 0 to n) in one beam and $n-i$ users in the other beam. Therefore $C_{m,n}$ can be written as:

$$C_{2,n} = \frac{1}{2^n} \sum_{i=0}^{n} \binom{n}{i} [1 - (1 - C_i)(1 - C_{(n-i)})] \qquad (13)$$

where the term in brackets immediately after the summation denotes the combination of selecting i users from a set of n.

To understand ((13)) consider the two beams in turn. If there are i users in the first beam, from ((12)), we know that the probability that the beam can receive the signal is C_i. In the second beam, the probability that the beam can receive the signal will be C_{n-i}. Therefore the probability that we can receive a message from either beam will be $1 - (1 - C_i)(1 - C_{(n-i)})$ and this accounts for the term in square brackets in ((13)). The remaining part of ((13)) takes account of the probability that there will be i users in the first beam and $n - i$ users in the second beam.

For the situation with 3 beams we get the expression

$$C_{3,n} = \frac{1}{3^n} \sum_{i=0}^{n} \binom{n}{i} \sum_{j=0}^{(n-i)} \binom{n-i}{j} [1 - (1 - C_i)(1 - C_j)(1 - C_{(n-i-j)})] \qquad (14)$$

In this expression we have i users in the first beam, j (from 0 to $n-i$) users in the second beam and $n-i-j$ users in the third beam. Therefore the probability of receiving a message is $1 - (1 - C_i)(1 - C_j)(1 - C_{(n-i-j)})$ while the probability of a given user distribution is given by the remaining terms. For larger m, we can use similar methods to generalize the formula.

Given $h(r), \sigma, b$ and the number of beams which are used to serve the cell, $C_{m,n}$ can be numerically evaluated for all n. In figure 2, the capture probability versus n is plotted for $b = 10dB$ and $\sigma = 6dB$ with different m for a uniform traffic density (i.e., $h(r) = 2r, r \in (0, 1]$).

As can be observed from the figure, the capture probability increases as the number of beams increases which quantifies the benefits of using smart antennas.

One interesting characteristic of the results is the dip in capture probability when there are 2 users ($n = 2$) for $m > 1$. This can be understood by considering the probability of there being only one user in any one of the beams since this corresponds to a high capture probability and will provide a trend. For example when there are 2 users in a cell, with two beams, the probability of one user being in a single beam is 1/2. However when n increases to three users the probability of a single user in one of the two beams rises to 2/3 so that the capture probability actually increases therefore creating the curve in figure 2 to rise and creating the dip. This trend continues for when there are more than two beams and is accounted for by the combination terms in the formulas ((13)) and ((14)).

It should also be noted that although there are diminishing returns with increasing m the returns remain worthwhile, in contrast to the diversity situation [4].

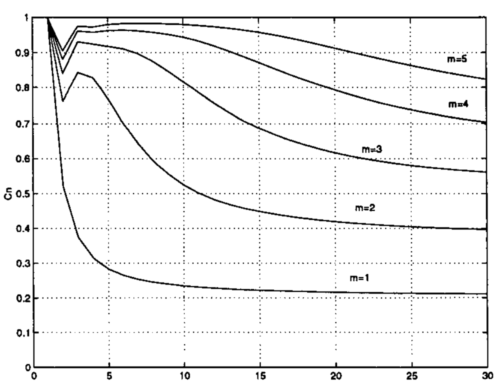

Fig. 2. Capture probabilities $C_{m,n}$ vs the number of colliding packets, n: $\eta = 4, \sigma = 6dB, b = 10dB$.

IV. MULTIPLE RECEIVERS

In the previous section, we considered the case where the base station has a single receiver. As a result, the base station can receive at most one message even though we have a multiple number of beams that allow the capture of a multiple number of messages. In this section, we report results on the effect of the base station having multiple receivers on the throughput of the system [4]. We assume that each base station's receiver can connect to any beam. Hence, if a beam successfully captures a message and there is a free receiver, then the receiver can connect to that beam to receive the message.

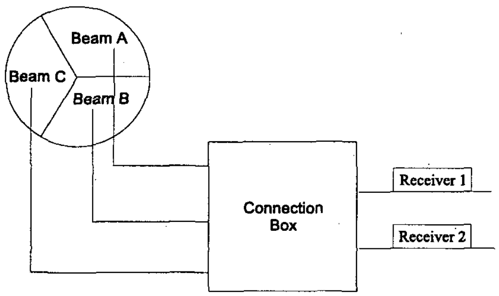

Fig. 3. An example of a base station with three antennas and two receivers.

Figure 3 illustrates the idea of connecting multiple antennas and multiple receivers. We assume that the base station has a connection box with some scanning mechanism which connects beams that successfully capture a message to a free receiver. In some cases it would be expected the number of receivers would be the same as the number of beams (it of course cannot be greater) and so the scanning mechanism would not be required.

When the base station has only one receiver, we use the capture probability $C_{m,n}$ as our performance measure as was shown in the previous section. However, when the base station has multiple receivers capable of simultaneously receiving multiple messages, we use throughput as our measure of performance [4].

Figure 4 illustrates the system throughput when there are two receivers with different number of beams. When compared to the case where the base station has only one receiver (please refer to figure 2), we can see that the throughput is higher when using two receivers. In addition, just as figure 2 illustrates, figure 4 illustrates the benefits of having multiple beams for increasing the capacity of the mobile environment.

Next, we fix the number of beams to five and increase the number of receivers from one to five and see the receiver's effect. The result is shown in figure 5. From the figure, we found that the throughput increases a lot when the number of receivers increase from one to two. But the improvement drops a lot when the number of receivers continue to increase and there is almost no improvement when the number of receivers increases from four to five

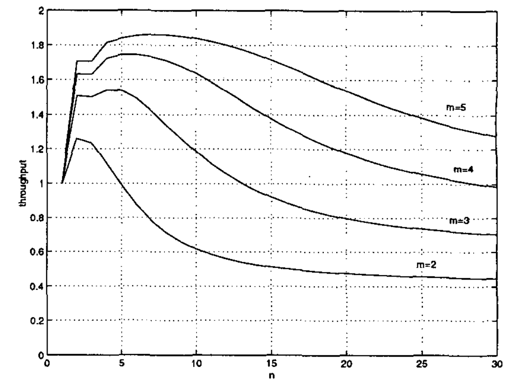

Fig. 4. Throughput for two receivers with different number of beams m.

especially when the number of users is large.

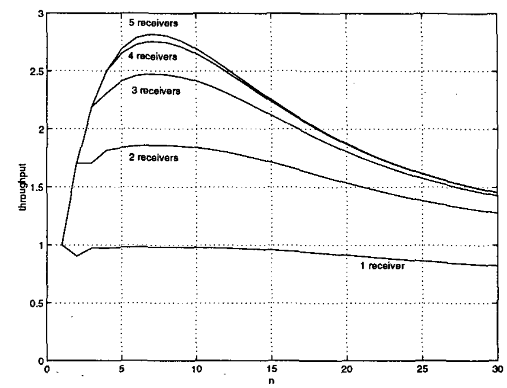

Fig. 5. Throughput when using five beams and different number of receivers, r.

We found the same trend when four beams are used with different number of receivers as shown in figure 6. The throughput also increases a lot when the number of receivers increase from one to two but nearly no effect when it increases from three to four.

From these results, we can conclude that the most cost-effective way is to use two receivers. More receivers will only have less additional benefits. Although the effect of increasing the number of beams also drops when the number is large, its dropping rate is much smaller that the case for receivers which justifies having more than two beams in our mobile environment.

V. STABILITY

From a stability view point it is important to determine the capture probability and throughput as the number of users tends to infinity. This is because if for example the capture probability drops to zero when the number of users exceed a certain number, our system will no longer

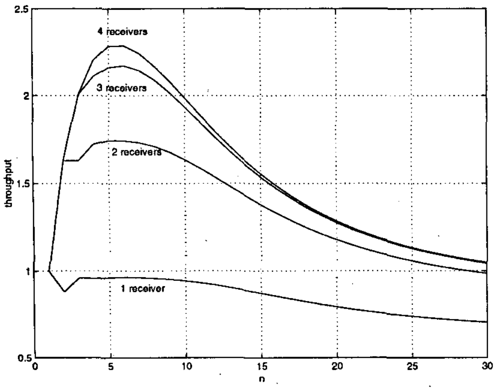

Fig. 6. Throughput when using four beams and different number of receivers, r.

capture any packets and becomes unstable.

figure 2 illustrates that using more beams will give a higher capture probability. Now we will try to show the stability of our proposed smart antenna when the number of users approaches infinity. The system will achieve a positive stable throughput if and only if $\lim_{n\to\infty} C_{m,n} > 0$. From [3], it was shown that if $\lim_{n\to\infty} h(r)/r = H_0$ exists, is finite, and strictly positive, then for any σ and $\eta = 4$,

$$C_\infty = \frac{2}{\pi\sqrt{b}} \tag{15}$$

Therefore in our system $C_\infty > 0$ will also approach this value and have a positive stable throughput. However for $m > 1$ we need to develop a new proof and here we consider the case of $C_{2,\infty}$ when there are two beams. Using the same conditions as [3] the existence of $\lim_{n\to\infty} h(r)/r = H_0$ (H_0 is finite, and strictly positive) with $\eta = 4$, we can write

$$\lim_{n\to\infty} C_{2,n} = \lim_{n\to\infty} \frac{1}{2^n} \sum_{i=0}^{n} \binom{n}{i} [1-(1-C_i)(1-C_{(n-i)})] \tag{16}$$

Substituting the bound ((15)) into ((16)) allows us to determine a bound on ((16)) as follows,

$$\Rightarrow \lim_{n\to\infty} C_{2,n} \geq \frac{1}{2^n} \sum_{i=0}^{n} \binom{n}{i} [1-(1-\frac{2}{\pi\sqrt{b}})(1-\frac{2}{\pi\sqrt{b}})]$$
$$\geq \frac{2}{\pi\sqrt{b}}(2-\frac{2}{\pi\sqrt{b}}) \tag{17}$$

This bound is larger than ((15)) and is also positive demonstrating the stability of our wireless system. In addition when our wireless system uses two beams, ((17)) indicates that $C_{2,\infty}$ is larger than C_∞ by a factor of $(2-\frac{2}{\pi\sqrt{b}})$ and will have higher capture probability in the limit. When we have more beams the system stability can also be shown using the same approach. In the case of multiple receivers the throughput will only be greater and therefore the minimum value will also be greater than zero demonstrating the system stability as well.

V. DISCUSSION AND CONCLUSIONS

In this paper we have demonstrated that the use of smart antennas significantly increases the capture probability and throughput of our system compared to conventional antennas. The marginal increase in capture probability for each additional beam decreases slowly with the number of beams. However the marginal increase in throughput decreases rapidly when more than 2 receivers are used. Therefore it would appear most effective to utilize two receivers and employ a switched beam antenna with several beams in our system.

REFERENCES

[1] K. C. Chen, "Medium Access Control of Wireless LANs for Mobile Computing", *IEEE Network*, pp. 50-63, 1994.

[2] O. Kubbar and H. T. Hussein, "Multiple Access Control Protocols for Wireless Networks: Problems, Definitions, and Design Objectives", *IEEE Communications Magazine*, Vol 35, No 11, pp93-99, November 1997.

[3] M. Zorzi and R. R. Rao, "Capture and Retransmission Control in Mobile Radio", *IEEE Journal on Selected Areas in Communications*, Vol 12, No 8, Oct 1994.

[4] M. Zorzi, "Mobile Radio slotted ALOHA with Capture and Diversity", *Wireless Networks*, Vol 1, No 2, June 1995.

[5] S. C. Swales and M. A. Beach and D. J. Edwards and J. P. McGeehan, "The Performance Enhancement of Multibeam Adaptive Base-Station Antennas for Cellular Land Mobile Radio Systems", *IEEE Veh. Technol.*, Vol VT-39, No 1, pp56-67, February 1990.

[6] R. Rheinschmitt and M. Tangemann, "Performance of Sectorised Spatial Multiplex Systems," *Proceedings of the IEEE Veh. Technol. Conference*, pp. 426-430, Atlanta, USA, 1996.

[7] G. V. Tsoulos, M. A. Beach and S. C. Swales, "On the Sensitivity of the Capacity Enhancement of a TDMA System with Adaptive Multibeam Antennas," *IEEE Veh. Technol.*, vol. VT-47, no. 1, pp. 165-169, May 1997.

[8] H. P. Lin, S. S. Jeng, I. Parra, G. Xu, W. J. Vogel and G. W. Torrence, "Experimental Studies of SDMA Schemes for Wireless Communications," *Proceedings of the Conference on Acoustics, Speech, and Signal Processing*, May 9-12, 1995, D etroit, USA, pp. 1760-1763.

[9] W.S. Au, C.T. Lea and R.D. Murch, "Improved Assessment of the Spectrum Efficiency of SDMA Systems", *IEEE Vehicular Technology Conference*, Ottawa, Canada, 18-21 May, 1998, pp 1999-2003.

[10] T. Kobayashi and I. Sasase, "Performance Analysis of PRMA with an Adaptive array", *Proc. PIMRC '97*, pp. 923-927, 1997.

[11] A. Sugihara, K. Enomoto and I. Sasase, "Throughput Performance of CSMA with an Adaptive Array", *Proc. PIMRC '95*, pp. 633-637, 1995.

[12] J-P M G Linnartz, "Exact Analysis of the Outage probability in multiple-user mobile radio", *IEEE Transactions on Communications*, Vol 40, No 1, pp 20-23, January 1992.

[13] R. Prasad and A. Kegel, "Improved Assessment of Interference Limits in Cellular Performance", *IEEE Transactions on Vehicular Technology*, Vol 40, No 2, pp 412-419, May 1991.